Introduction to the Gnu Linear Programming Kit

Optimizing financial and industry models with GLPK

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Introduction
Gnu Linear Programming Kit

- organized as a callable library
Gnu Linear Programming Kit

- organized as a callable library
- written in ANSI C
Gnu Linear Programming Kit

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**objectives**
- solve linear programming problems
- solve mixed integer programming problems
- solve some other related problems
- organized as a callable library
- written in ANSI C

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- solve linear programming problems
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- current version: 4.20
Gnu Linear Programming Kit

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- objectives
  - solve linear programming problems
  - solve mixed integer programming problems
  - solve some other related problems

- current version: 4.20
- home page: http://www.gnu.org/software/glpk/glpk.html
Characteristics
methods available for solving problems

- revised simplex algorithm
methods available for solving problems

- revised simplex algorithm
- primal-dual interior point algorithm
methods available for solving problems

- revised simplex algorithm
- primal-dual interior point algorithm
- branch & bound algorithm
Solvers

- standalone solver
  - glpsol
  - default solver in GLPK package
solvers

- standalone solver
  - glpsol
    - default solver in GLPK package

- other options
  - glpkmex - the Matlab MEX Interface of GLPK
  - DELI - interface for Delphi
  - JNI interface
Languages for describing problems

- Fixed MPS format (glpsol's default language)
Languages for describing problems

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- GNU LP format
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- Free MPS format
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- GNU MathProg modeling language
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- GNU MathProg modeling language

This tutorial focuses on the Gnu MathProg.
Learn by example
Two-var problem: Giapetto’s Woodcarving

From Winston, Wayne L. - Operations Research

- **two types of wooden toys: soldiers and trains**
- **soldier**: sells for £27, uses £10 worth of raw materials, increases variable labor and overhead costs by £14
- **train**: sells for £21, uses £9 worth of raw materials, increases variable labor and overhead costs by £10
- **soldier**: requires 2 hours of finishing labor and 1 hour of carpentry labor
- **train**: requires 1 hour of finishing labor and 1 hour of carpentry labor
- Maximum of 100 finishing hours and 80 carpentry hours is available weekly
- **Weekly demand**: trains (unlimited), soldiers (40)

Giapetto wants to maximize weekly profits (revenues - costs)
Two-var problem: Giapetto’s Woodcarving

Mathematical formulation

- **Decision variables**
  \( x_1 \): Soldiers produced each week
  \( x_2 \): Trains produced each week

- **Objective function**
  \[ \max z = (27x_1 + 21x_2) - (10x_1 + 9x_2) - (14x_1 + 10x_2) = 3x_1 + 2x_2 \]

- **Constraints**
  \[ 2x_1 + x_2 \leq 100 \text{ (finishing constraint)} \]
  \[ x_1 + x_2 \leq 80 \text{ (carpentry constraint)} \]
  \[ x_1 \leq 40 \text{ (constraint on demand for soldiers)} \]
  \[ x_1 \geq 0, \ x_2 \geq 0 \text{ (sign constraints)} \]
Two-var problem: Giapetto’s Woodcarving

Analyze glpsol results

- rows
  - St
  - Activity
  - Lower bound
  - Upper bound
  - Marginal
Two-var problem: Giapetto’s Woodcarving

Enhancing the model

- Parameters
- Data section
- Summation
The diet problem

From Winston, Wayne L. - Operations Research

Satisfy my daily nutritional requirements at minimum cost

<table>
<thead>
<tr>
<th>Type of food</th>
<th>Cost per unity</th>
<th>Daily needs</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brownie</td>
<td>£0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chocolate ice cream</td>
<td>£0.2 (scoop)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cole</td>
<td>£0.3 (bottle)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pineapple cheesecake</td>
<td>£0.8 (piece)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type of food</th>
<th>Calories</th>
<th>Chocolate(oz)</th>
<th>Sugar(oz)</th>
<th>Fat(oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brownie</td>
<td>400</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Chocolate ice cream (scoop)</td>
<td>200</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Cole (1 bottle)</td>
<td>150</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Pineapple cheesecake (1 piece)</td>
<td>500</td>
<td>0</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
The diet problem

Mathematical formulation

- **Decision variables**
  \[ x_1 : \text{number of brownies eaten daily} \]
  \[ x_2 : \text{number of scoops of chocolate ice cream eaten daily} \]
  \[ x_3 : \text{bottles of cola trunk daily} \]
  \[ x_4 : \text{pieces of pineapple cheesecake eaten daily} \]

- **Objective function**
  \[
  \min z = 50x_1 + 20x_2 + 30x_3 + 80x_4
  \]

- **Constraints**
  \[
  400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 500 \quad (Calorie constraint)
  \]
  \[
  3x_1 + 2x_2 \geq 6 \quad (Chocolate constraint)
  \]
  \[
  2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 10 \quad (Sugar constraint)
  \]
  \[
  2x_1 + 4x_2 + x_3 + 5x_4 \geq 8 \quad (Fat constraint)
  \]
  \[
  x_i \geq 0, \quad \forall i \in \{1, \ldots, 4\}
  \]
The diet problem

- Enhancements
  - 2-dimensional table
The diet problem

- Enhancements
  - 2-dimensional table
- Analyze the results interactively
Short term financial problem

Modified from Winston, Wayne L. - Operations Research

- **Semicond manufactures tape recorders and radios**
- costs, and selling price are given in one of the tables
- available raw material sufficient to manufacture 100 tape recorders and 100 radios
- balance sheet is shown in one of the tables
- asset-liability ratio is 20000/10000 = 2
- Demand: unlimited
- Semicond will collect £2000 in accounts receivable
- Semicond must pay off £1000 of the outstanding loan and a monthly rent of £1000
- January 1, 2008: receive a shipment of raw material worth £2000
- cash balance must be at least £4000
- current ratio must be at least 2

What should Semicond produce on December?
# Short term financial problem

## Problem data

<table>
<thead>
<tr>
<th></th>
<th>Tape recorder</th>
<th>Radio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>£130</td>
<td>£110</td>
</tr>
<tr>
<td>Labor cost</td>
<td>£50</td>
<td>£35</td>
</tr>
<tr>
<td>Raw material cost</td>
<td>£30</td>
<td>£40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Asserts</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>£10,000</td>
<td></td>
</tr>
<tr>
<td>Accounts receivable</td>
<td>£3,000</td>
<td></td>
</tr>
<tr>
<td>Inventory outstanding</td>
<td>£7,000</td>
<td></td>
</tr>
<tr>
<td>Bank loan</td>
<td></td>
<td>£10,000</td>
</tr>
</tbody>
</table>
Mathematical formulation

- **Decision variables**
  
  \( x_1 \): tapes produced  
  \( x_2 \): radios produced

- **Objective function**

  \[
  \text{max } z = 50x_1 + 35x_2 \tag{1}
  \]

- **Constraints**

  \[
  \begin{align*}
  x_1 & \leq 100 \quad \text{(tape constraint)} \\
  x_2 & \leq 100 \quad \text{(radio constraint)} \\
  50x_1 + 35x_2 & \leq 6000 \quad \text{(cash position constraint)} \\
  50x_1 + 35x_2 & \geq 2000 \quad \text{(ratio constraint)} \\
  x_1 & \geq 0, \quad x_2 \geq 0
  \end{align*}
  \]
Short term financial problem

Let's modify the problem a little bit...
Short term financial problem

- Let's modify the problem a little bit...
- Multiple solution problem!
Brute production process

Modified from Winston, Wayne L. - Operations Research

- Rylon Corporation manufactures Brute and Chanelle perfumes
- raw material: purchased for £3 per pound
- processing 1lb of raw material: 1 hour of laboratory time
- each pound of raw-material: 3oz of Regular Brute Perfume and 4oz of Regular Chanelle Perfume
- Regular Brute sells for £7/oz and Regular Chanelle for £6/oz
- reprocessing: Luxury Brute, sold at £18/oz, and Luxury Chanelle, sold at £14/oz
- Each oz of Regular Brute processed further: additional of 3 hours of lab time and £4 processing cost, yields 1oz of Luxury Brute
- Each oz of regular Chanelle processed further: additional 2 hours of lab time and £4 processing cost, yields 1oz of Luxury Chanelle
- yearly: 6000 hours of lab time available and can purchase up to 4000lb of raw material

Maximize Rylon’s profit
Brute production process

Mathematical formulation

- Decision variables
  
  $x_1$: No. of oz of Regular Brute sold annually
  
  $x_2$: No. of oz of Luxury Brute sold annually
  
  $x_3$: No. of oz of Regular Chanelle sold annually
  
  $x_4$: No. of oz of Luxury Chanelle sold annually
  
  $x_5$: No. of pounds of raw material purchased annually

- Objective function

  \[
  \max \; z = 7x_1 + 14x_2 + 6x_3 + 10x_4 - 3x_5
  \]
Brute production process

- Constraints

(1) \[ x_5 \leq 4000 \text{ (raw material)} \]

(2) \[ 3x_2 + 2x_4 + x_5 \leq 6000 \text{ (lab hours)} \]

(3) \[ x_1 + x_2 - 3x_5 = 0 \text{ (mass conservation)} \]

(4) \[ x_3 + x_4 - 4x_5 = 0 \text{ (mass conservation)} \]

(5) \[ x_i \geq 0, \ \forall i \in \{1,..,5\} \]
Brute production process

- Let's modify it a little bit: no mass conservation
Brute production process

- Let's modify it a little bit: no mass conservation
- What should the result be?
Brute production process

- Let’s modify it a little bit: no mass conservation
- What should the result be?
- Unbounded problem!
Multi-period investments

From Winston, Wayne L. - Operations Research

- Finco needs an investment strategy for the next three years
- There are five investments available
- Cash flow for each invested £1 is in the table below
- At most £75000 should be placed in any investment
- Finco can earn 8% per year with money market funds with uninvested cash
- Finco can not borrow money
- Finco has £100000 available to invest

<table>
<thead>
<tr>
<th>Investment</th>
<th>Time 0</th>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-1</td>
<td>+0.5</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>-1</td>
<td>+0.5</td>
<td>+1</td>
</tr>
<tr>
<td>C</td>
<td>-1</td>
<td>+1.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>+1.9</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>+1.5</td>
</tr>
</tbody>
</table>

Maximize Finco’s cash by the end of the third year
Multi-period investments

Mathematical formulation

- **Decision variables**
  \[ x_1: \text{pounds invested in investment A} \]
  \[ x_2: \text{pounds invested in investment B} \]
  \[ x_3: \text{pounds invested in investment C} \]
  \[ x_4: \text{pounds invested in investment D} \]
  \[ x_5: \text{pounds invested in investment E} \]
  \[ s_t: \text{pounds invested in money market funds at time } t, \ t \in \{0, 1, 2\} \]

- **Objective function**

  \[
  \max \quad z = x_2 + 1.9x_4 + 1.5x_5 + 1.08s_2
  \]
Multi-period investments

## Constraints

1. \[ x_1 + x_3 + x_4 + s_0 = 100000 \] (investment at time 0)
2. \[ 0.5x_1 + 1.2x_3 + 1.08s_0 = x_2 + s_1 \] (investment at time 1)
3. \[ x_1 + 0.5x_2 + 1.08s_1 = x_5 + s_2 \] (investment at time 2)
4. \[ x_i \leq 75000, \ \forall i \in \{1, \ldots, 5\} \] (maximum single investment)
5. \[ x_i \geq 0, \ \forall i \in \{1, \ldots, 5\} \] (sign constraint)
6. \[ s_t \geq 0, \ \forall t \in \{0, 1, 2\} \] (sign constraint)
Multi-period investments

- Let’s check out the results!
Oil blending

From Winston, Wayne L. - Operations Research

- Sunco manufactures three types of gasoline
- Each gasoline type is a blending from three types of crude
- Price per barrel of gasoline and crude are given below
- At most 5000 barrels of each crude is purchased daily
- Octane rating and sulfur level for each gasoline is given below
- Octane rating and sulfur level of each crude is given below
- It costs £4 to refine each barrel of crude
- Refinery capacity is 14000 barrels per day
- Demand for gasoline is given below
- Gasoline advertisement: increase of 10 barrels per £1 spent

<table>
<thead>
<tr>
<th>Gas</th>
<th>Demand</th>
<th>Sells</th>
<th>Octane</th>
<th>Sulfur (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3000</td>
<td>£70</td>
<td>10</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>2000</td>
<td>£60</td>
<td>8</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>£50</td>
<td>6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Crude</th>
<th>Bought</th>
<th>Octane</th>
<th>Sulfur (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>£45</td>
<td>12</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>£35</td>
<td>6</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>£25</td>
<td>8</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Maximize Sunco’s profit
Oil blending

Mathematical formulation

- **Decision variables**
  \[ x_{ij} : \text{barrels of crude oil } i \text{ used daily to produce gas } j; \ i \in \{1,2,3\}, \ j \in \{1,2,3\} \]
  \[ a_j : \text{pounds spent daily on advertising gas } j, \ j \in \{1,2,3\} \]

- **Objective function**
  \[
  \max z = 21x_{11} + 11x_{12} + x_{13} + 31x_{21} + 21x_{22} - 11x_{23} + \\
  + 41x_{31} + 31x_{32} + 21x_{33} - a_1 - a_2 - a_3
  \]
Oil blending

### Constraints

1. \( x_{11} + x_{21} + x_{31} = 3000 + 10a_1 \) (gas 1 daily demand)
2. \( x_{12} + x_{22} + x_{32} = 2000 + 10a_2 \) (gas 2 daily demand)
3. \( x_{13} + x_{23} + x_{33} = 1000 + 10a_3 \) (gas 3 daily demand)
4. \( x_{11} + x_{12} + x_{13} \leq 5000 \) (crude 1 max daily purchase)
5. \( x_{21} + x_{22} + x_{23} \leq 5000 \) (crude 2 max daily purchase)
6. \( x_{31} + x_{32} + x_{33} \leq 5000 \) (crude 3 max daily purchase)
7. \( \sum_{ij} x_{ij} \leq 14000, \ \forall i \in \{1, 2, 3\}, \ \forall j \in \{1, 2, 3\} \) (refinery capacity)
8. \( 2x_{11} - 4x_{21} - 2x_{31} \geq 0 \) (gas 1 octane level)
9. \( 4x_{12} - 2x_{22} \geq 0 \) (gas 2 octane level)
10. \( 6x_{13} + 2x_{33} \geq 0 \) (gas 3 octane level: redundant)
11. \( -0.005x_{11} + 0.01x_{21} + 0.02x_{31} \leq 0 \) (gas 1 sulfur level)
12. \( -0.015x_{12} + 0.01x_{32} \leq 0 \) (gas 2 sulfur level)
13. \( -0.005x_{13} + 0.01x_{23} + 0.02x_{33} \leq 0 \) (gas 3 sulfur level)
Oil blending

- Show me the results!
Final words
Discussion lists
Where to go for help?

- Discussion lists
  - Main discussion list: help, development, porting, enhancement request
    - help-glpk@gnu.org
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    - help-glpk@gnu.org

- Bugs
  - bug-glpk@gnu.org
GLPK is maintained by Andrew Makhorin (mao@mai2.rcnet.ru)
Questions
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- Open Source Community
- Linuxconf Europe 2007 organizers

Thank you!
Contact

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