Introduction to the Gnu Linear Programming Kit

Optimizing financial and industry models with GLPK

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Introduction

Gnu Linear Programming Kit

Introduction
- Gnu Linear Programming Kit

Characteristics

Learn by example

Final words

- organized as a callable library

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  - solve linear programming problems
  - solve mixed integer programming problems
  - solve some other related problems

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- home page: http://www.gnu.org/software/glpk/glpk.html

Characteristics

methods available for solving problems

- revised simplex algorithm

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- revised simplex algorithm
- primal-dual interior point algorithm

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- revised simplex algorithm
- primal-dual interior point algorithm
- branch & bound algorithm

Solvers

- standalone solver
  - glpsol
    - default solver in GLPK package

Solvers

- standalone solver
  - glpsol
    - default solver in GLPK package
- other options
  - glpkmex - the Matlab MEX Interface of GLPK
  - DELI - interface for Delphi
  - JNI interface

Languages for describing problems

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- **GNU MathProg modeling language**

This tutorial focuses on the Gnu MathProg.

Learn by example

Two-var problem: Giapetto’s Woodcarving

From Winston, Wayne L. - Operations Research

- *two types of wooden toys: soldiers and trains*
- *soldier: sells for £27, uses £10 worth of raw materials, increases variable labor and overhead costs by £14*
- *train: sells for £21, uses £9 worth of raw materials, increases variable labor and overhead costs by £10*
- *soldier: requires 2 hours of finishing labor and 1 hour of carpentry labor*
- *train: requires 1 hour of finishing labor and 1 hour of carpentry labor*
- *Maximum of 100 finishing hours and 80 carpentry hours is available weekly*
- *Weekly demand: trains (unlimited), soldiers (40)*

Giapetto wants to maximize weekly profits (revenues - costs)

Two-var problem: Giapetto's Woodcarving

Mathematical formulation

- Decision variables
  - x_1: Soldiers produced each week
  - x_2: Trains produced each week
- Objective function

(1) $max \;\; z = (27x_1 + 21x_2) - (10x_1 + 9x_2) - (14x_1 + 10x_2) = 3x_1 + 2x_2$

- Constraints

(2) $2x_1 + x_2 \leq 100 \;\; (finishing\ constraint)$

(3) $x_1 + x_2 \leq 80 \;\; (carpentry\ \ constraint)$

(4) $x_1 \leq 40 \;\; (constraint\ on\ demand\ for\ soldiers)$

(5) $x_1 \geq 0, \;\; x_2 \geq 0 \;\; (sign\ contraints)$

Two-var problem: Giapetto's Woodcarving

Analyze glpsol results

- rows
  - St
  - Activity
  - Lower bound
  - Upper bound
  - Marginal

Two-var problem: Giapetto’s Woodcarving

Enhancing the model

- Parameters
- Data section
- Summation

The diet problem

From Winston, Wayne L. - Operations Research

Satisfy my daily nutritional requirements at minimum cost

Type of food	Cost per unity
Brownie	£0.5
Chocolate ice cream	£0.2 (scoop)
Cole	£0.3 (bottle)
Pineapple cheesecake	£0.8 (piece)

Daily needs	Amount
Calories	500
Chocolate	6 oz
Sugar	10 oz
Fat	8 oz

Type of food	Calories	Chocolate(oz)	Sugar(oz)	Fat(oz)
Brownie	400	3	2	2
Chocolate ice cream (scoop)	200	2	2	4
Cole (1 bottle)	150	0	4	1
Pineapple cheesecake (1 piece)	500	0	4	5

The diet problem

Mathematical formulation

- Decision variables

  x_1: number of brownies eaten daily

  x_2: number of scoops of chocolate ice cream eaten daily

  x_3: bottles of cola trunk daily

  x_4: pieces of pineapple cheesecake eaten daily

- Objective function

(1) $$min \ \ z = 50x_1 + 20x_2 + 30x_3 + 80x_4$$

- Constraints

(2) $$400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 500 \ \ (Calorie\ constraint)$$

(3) $$3x_1 + 2x_2 \geq 6 \ \ (Chocolate\ constraint)$$

(4) $$2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 10 \ \ (Sugar\ constraint)$$

(5) $$2x_1 + 4x_2 + x_3 + 5x_4 \geq 8 \ \ (Fat\ constraint)$$

(6) $$x_i \geq 0, \ \ \forall i \in \{1, .., 4\}$$

The diet problem

- Enhancements
  - 2-dimensional table

The diet problem

- Enhancements
  - 2-dimensional table
- Analyze the results interactively

Short term financial problem

Modified from Winston, Wayne L. - Operations Research

- *Semicond manufactures tape recorders and radios*
- *costs, and selling price are given in one of the tables*
- *available raw material sufficient to manufacture 100 tape recorders and 100 radios*
- *balance sheet is shown in one of the tables*
- *asset-liability ratio is 20000/10000 = 2*
- *Demand: unlimited*
- *Semicond will collect £2000 in accounts receivable*
- *Semicond must pay off £1000 of the outstanding loan and a monthly rent of £1000*
- *January 1, 2008: receive a shipment of raw material worth £2000*
- *cash balance must be at least £4000*
- *current ratio must be at least 2*

What should Semicond produce on December?

Short term financial problem

Problem data

	Tape recorder	Radio
Selling price	£130	£110
Labor cost	£50	£35
Raw material cost	£30	£40

	Asserts	Liability
Cash	£10.000	
Accounts receivable	£3.000	
Inventory outstanding	£7.000	
Bank loan		£10.000

Short term financial problem

Mathematical formulation

- Decision variables

  x_1: tapes produced

  x_2: radios produced

- Objective function

(1) $$max \;\; z = 50x_1 + 35x_2$$

- Constraints

(2) $$x_1 \leq 100 \;\; (tape\ constraint)$$

(3) $$x_2 \leq 100 \;\; (radio\ constraint)$$

(4) $$50x_1 + 35x_2 \leq 6000 \;\; (cash\ position\ constraint)$$

(5) $$50x_1 + 35x_2 \geq 2000 \;\; (ratio\ constraint)$$

(6) $$x_1 \geq 0, \;\; x_2 \geq 0$$

Short term financial problem

- Let's modify the problem a little bit...

Short term financial problem

- Let's modify the problem a little bit...
- Multiple solution problem!

Brute production process

Modified from Winston, Wayne L. - Operations Research

- *Rylon Corporation manufactures Brute and Chanelle perfumes*
- *raw material: purchased for £3 per pound*
- *processing 1lb of raw material: 1 hour of laboratory time*
- *each pound of raw-material: 3oz of Regular Brute Perfume and 4oz of Regular Chanelle Perfume*
- *Regular Brute sells for £7/oz and Regular Chanelle for £6/oz*
- *reprocessing: Luxury Brute, sold at £18/oz, and Luxury Chanelle, sold at £14/oz*
- *Each oz of Regular Brute processed further: additional of 3 hours of lab time and £4 processing cost, yields 1oz of Luxury Brute*
- *Each oz of regular Chanelle processed further: additional 2 hours of lab time and £4 processing cost, yields 1oz of Luxury Chanelle*
- *yearly: 6000 hours of lab time available and can purchase up to 4000lb of raw material*

Maximize Rylon's profit

Brute production process

Mathematical formulation

- Decision variables

  x_1: No. of oz of Regular Brute sold annually
  x_2: No. of oz of Luxury Brute sold annually
  x_3: No. of oz of Regular Chanelle sold annually
  x_4: No. of oz of Luxury Chanelle sold annually
  x_5: No. of pounds of raw material purchased annually

- Objective function

(1) $$max\ \ z = 7x_1 + 14x_2 + 6x_3 + 10x_4 - 3x_5$$

Brute production process

- Constraints

(1) $$x_5 \leq 4000 \;\; (raw\ material)$$

(2) $$3x_2 + 2x_4 + x_5 \leq 6000 \;\; (lab\ hours)$$

(3) $$x_1 + x_2 - 3x_5 = 0 \;\; (mass\ conservation)$$

(4) $$x_3 + x_4 - 4x_5 = 0 \;\; (mass\ conservation)$$

(5) $$x_i \geq 0, \;\; \forall i \in \{1, .., 5\}$$

Brute production process

- Let's modify it a little bit: no mass conservation

Brute production process

- Let's modify it a little bit: no mass conservation
- What should the result be?

Brute production process

- Let's modify it a little bit: no mass conservation
- What should the result be?
- Unbounded problem!

Multi-period investments

From Winston, Wayne L. - Operations Research

- *Finco needs an investment strategy for the next three years*
- *There are five investments available*
- *Cash flow for each invested £1 is in the table below*
- *At most £75000 should be placed in any investment*
- *Finco can earn 8% per year with money market funds with uninvested cash*
- *Finco can not borrow money*
- *Finco has £100000 available to invest*

Investment	Time 0	Time 1	Time 2	Time 3
A	-1	+0.5	+1	0
B	0	-1	+0.5	+1
C	-1	+1.2	0	0
D	-1	0	0	+1.9
E	0	0	-1	+1.5

Maximize Finco's cash by the end of the third year

Multi-period investments

Mathematical formulation

- Decision variables

  x_1: pounds invested in investment A
  x_2: pounds invested in investment B
  x_3: pounds invested in investment C
  x_4: pounds invested in investment D
  x_5: pounds invested in investment E
  s_t: pounds invested in money market funds at time t, $t \in \{0, 1, 2\}$

- Objective function

$$max \;\; z = x_2 + 1.9x_4 + 1.5x_5 + 1.08s_2 \tag{1}$$

Multi-period investments

Constraints

(1) $$x_1 + x_3 + x_4 + s_0 = 100000 \ \ (investment\ at\ time\ 0)$$

(2) $$0.5x_1 + 1.2x_3 + 1.08s_0 = x_2 + s_1 \ \ (investment\ at\ time\ 1)$$

(3) $$x_1 + 0.5x_2 + 1.08s_1 = x_5 + s_2 \ \ (investment\ at\ time\ 2)$$

(4) $$x_i \leq 75000, \ \ \forall i \in \{1, .., 5\} \ \ (maximum\ single\ investment)$$

(5) $$x_i \geq 0, \ \ \forall i \in \{1, .., 5\} \ \ (sign\ constraint)$$

(6) $$s_t \geq 0, \ \ \forall t \in \{0, 1, 2\} \ \ (sign\ constraint)$$

Multi-period investments

- Let's check out the results!

Oil blending

From Winston, Wayne L. - Operations Research

- *Sunco manufactures three types of gasoline*
- *Each gasoline type is a blending from three types of crude*
- *Price per barrel of gasoline and crude are given below*
- *At most 5000 barrels of each crude is purchased daily*
- *Octane rating and sulfur level for each gasoline is given below*
- *Octane rating and sulfur level of each crude is given below*
- *It costs £4 to refine each barrel of crude*
- *Refinery capacity is 14000 barrels per day*
- *Demand for gasoline is given below*
- *Gasoline advertisement: increase of 10 barrels per £1 spent*

Gas	Demand	Sells	Octane	Sulfur (%)
1	3000	£70	10	1.0
2	2000	£60	8	2.0
3	1000	£50	6	1.0

Crude	Bought	Octane	Sulfur(%)
1	£45	12	0.5
2	£35	6	2.0
3	£25	8	3.0

Maximize Sunco's profit

Oil blending

Mathematical formulation

- Decision variables

  x_{ij} : barrels of crude oil i used daily to produce gas j; $i \in \{1, 2, 3\}$, $j \in \{1, 2, 3\}$

  a_j : pounds spent daily on advertising gas j, $j \in \{1, 2, 3\}$

- Objective function

$$max \;\; z = 21x_{11} + 11x_{12} + x_{13} + 31x_{21} + 21x_{22} - 11x_{23} + \\ + 41x_{31} + 31x_{32} + 21x_{33} - a_1 - a_2 - a_3 \qquad (1)$$

Oil blending

Introduction

Characteristics

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- The diet problem
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- Multi-period investments
- Oil blending

Final words

Constraints

(1) $$x_{11} + x_{21} + x_{31} = 3000 + 10a_1 \ \ (gas\ 1\ daily\ demand)$$

(2) $$x_{12} + x_{22} + x_{32} = 2000 + 10a_2 \ \ (gas\ 2\ daily\ demand)$$

(3) $$x_{13} + x_{23} + x_{33} = 1000 + 10a_3 \ \ (gas\ 3\ daily\ demand)$$

(4) $$x_{11} + x_{12} + x_{13} \leq 5000 \ \ (crude\ 1\ max\ daily\ purchase)$$

(5) $$x_{21} + x_{22} + x_{23} \leq 5000 \ \ (crude\ 2\ max\ daily\ purchase)$$

(6) $$x_{31} + x_{32} + x_{33} \leq 5000 \ \ (crude\ 3\ max\ daily\ purchase)$$

(7) $$\sum_{ij} x_{ij} \leq 14000, \ \ \forall i \in \{1,2,3\}, \ \ \forall j \in \{1,2,3\} \ \ (refinery\ capacity)$$

(8) $$2x_{11} - 4x_{21} - 2x_{31} \geq 0 \ \ (gas\ 1\ octane\ level)$$

(9) $$4x_{12} - 2x_{22} \geq 0 \ \ (gas\ 2\ octane\ level)$$

(10) $$6x_{13} + 2x_{33} \geq 0 \ \ (gas\ 3\ octane\ level:\ \ redundant)$$

(11) $$-0.005x_{11} + 0.01x_{21} + 0.02x_{31} \leq 0 \ \ (gas\ 1\ sulfur\ level)$$

(12) $$-0.015x_{12} + 0.01x_{32} \leq 0 \ \ (gas\ 2\ sulfur\ level)$$

(13) $$-0.005x_{13} + 0.01x_{23} + 0.02x_{33} \leq 0 \ \ (gas\ 3\ sulfur\ level)$$

Oil blending

- Show me the results!

Final words

Where to go for help?

- Discussion lists

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    - help-glpk@gnu.org
  - Bugs
    - bug-glpk@gnu.org

Maintainer

GLPK is maintained by Andrew Makhorin (mao@mai2.rcnet.ru)

Questions

?

Acknowledgments

- Open Source Community
- Linuxconf Europe 2007 organizers

Thank you!

Contact

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